



# The optimal inventory policies under permissible delay in payments depending on the ordering quantity

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## Abstract

This paper deals with the problem of determining the economic order quantity under conditions of permissible delay in payments. The delay in payments depends on the quantity ordered. When the order quantity is less than the quantity at which the delay in payments is permitted, the payment for the item must be made immediately. Otherwise, the fixed trade credit period is permitted. The minimization of the total variable cost per unit of time is taken as the objective function. An algorithm to determine the economic order quantity is developed. The results obtained in this paper generalize some already published results.

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## 1. Introduction

The classical EOQ model assumes that the retailer's capital is unconstrained and the retailer must be paid for the items as soon as the items were received. However, the supplier may offer the retailer a delay period, that is the trade credit period, in settling the accounts. The effect of supplier credit policies on optimal order quantity has received the attention of many researchers; see Aggarwal and Jaggi (1995), Chang and Dye (2001), Chang et al. (2001), Chen and Chuang

(1999), Chu et al. (1998), Chung (1998a, b, 2000), Goyal (1985), Jamal et al. (1997, 2000), Khouja and Mehrez (1996), Liao et al. (2000), Sarker et al. (2000a,b) and Shah and Shah (1998). Recently, Arcelus et al. (2003) modeled the retailer's profit-maximizing retail promotion strategy, when confronted with a vendor's trade promotion offer of credit and/or price discount on the purchase of regular or perishable merchandise. Abad and Jaggi (2003) developed a joint approach to determine for the seller the optimal unit price and the length of the credit period when end demand is price sensitive. Chung and Huang (2003) investigated this issue within EPQ framework and developed an efficient solution procedure to determine the optimal cycle time for the retailer. Salameh et al.

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(2003) extended this issue to continuous review inventory model.

In 1996, Khouja and Mehrez investigated the effect of supplier credit policies on the optimal order quantity within the economic order quantity framework. The supplier credit policies fall into two categories: (1) supplier credit policies where credit terms are independent of the order quantity and (2) supplier credit policies where credit terms are linked to the order quantity. In the latter case, suppliers use favorable credit terms to encourage customers to order large quantities. In other words, the favorable credit terms apply only at large order quantities and are used in place of quantity discounts. Four supplier credit policies are introduced in Khouja and Mehrez (1996). The credit terms of the first two policies are independent of the order quantity. The last two credit policies link credit terms to the order quantity. The purpose of this paper is to discuss a few different credit policies and extend the work of Khouja and Mehrez (1996).

## 2. Model formulation

In this section, we want to develop the inventory model under permissible delay in payments to take the order quantity into account. When the order quantity is less than the fixed quantity at which the delay in payments is permitted ( $Q < W$ ), the payment for the items must be made immediately. Otherwise, the fixed trade credit period  $M$  is permitted. In addition, this paper tries to consider some alternations to move capital to match the policy of enterprise. We assume that the retailer will borrow 100% purchasing cost from the bank to pay off the account and the retailer does not return money to the bank until the end of the inventory cycle when the retailer needs cash to pay off the account. The following notation and assumptions will be used throughout.

### 2.1. Notation

$Q$	order quantity
$D$	annual demand
$W$	the fixed quantity at which the delay in payments is permitted

$A$	cost of placing one order
$c$	unit purchasing price per item
$s$	unit selling price per item
$h$	unit stock holding cost per item per year excluding interest charges
$I_e$	interest rate that can be earned per \$ per year
$I_p$	interest rate charged per \$ investment in inventory per year
$M$	trade credit period in years
$T$	the cycle time in years
$T^*$	the optimal cycle time of $TVC(T)$

### 2.2. Assumptions

- (1) Demand rate is known and constant.
- (2) Shortages are not allowed.
- (3) Time period is infinite.
- (4) Replenishments are instantaneous with a known and constant lead time.
- (5) When the retailer must pay the amount of purchasing cost to the supplier, the retailer will borrow 100% purchasing cost from the bank to pay off the account with rate  $I_p$ . When  $T \geq M$ , the retailer returns money to the bank at the end of the inventory cycle. However, when  $T \leq M$ , the retailer returns money to the bank at  $T = M$ .
- (6) If the credit period is shorter than the cycle length, the retailer can sell the items, accumulate sales revenue and earn interest with rate  $I_e$  throughout the inventory cycle.
- (7)  $s \geq c$  and  $I_p \geq I_e$ .

The total annual variable cost consists of the following elements. Two situations may arise. (I)  $W/D \leq M$  and (II)  $W/D > M$ .

- (I) Suppose that  $W/D \leq M$ .
  - (1) Annual ordering cost =  $\frac{A}{T}$ .
  - (2) Annual stock holding cost (excluding interest charges) =  $\frac{DTh}{2}$ .
  - (3) There are three cases to occur in interest payable per year.

Case I:  $0 < T < W/D$ , shown in Fig. 1.

In this case, the retailer must pay the amount of purchasing cost as soon as the items were received since  $Q < W$ . According to assumption (5), the retailer will borrow 100% purchasing cost,  $cDT$ ,

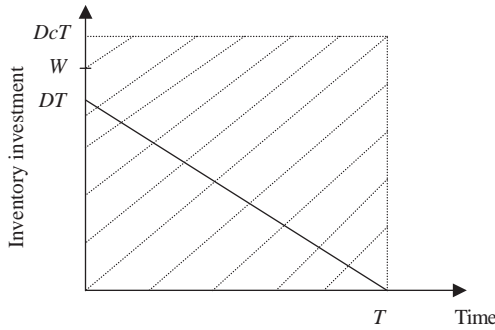


Fig. 1. The total accumulation of interest payable when  $0 < T < W/D$ .

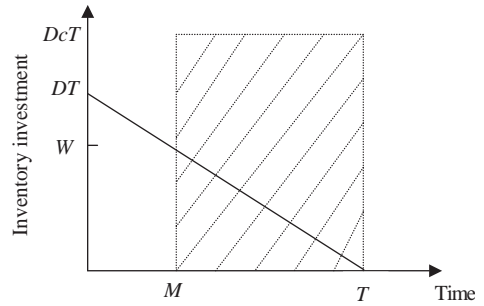


Fig. 2. The total accumulation of interest payable when  $M \leq T$ .

from the bank to pay off the account with rate  $I_p$  and return money to the bank at the end of the inventory cycle. So, the loan period is  $T$ .

Interest payable per cycle =  $cI_pDT^2$ .

Interest payable per year =  $cI_pDT$ .

Case II:  $W/D \leq T \leq M$ .

In this case, the fixed trade credit period  $M$  is permitted since  $Q \geq W$ . And the retailer do not need to loan anything from the bank at the end of credit period because the cycle length is not longer than the credit period. So, Interest payable per year = 0.

Case III:  $M \leq T$ , shown in Fig. 2.

In this case, the fixed trade credit period  $M$  is permitted since  $Q \geq W$ . According to assumption (5), the retailer will borrow 100% purchasing cost,  $cDT$ , from the bank to pay off the account with rate  $I_p$  and return money to the bank at the end of the inventory cycle. So the loan period is  $(T - M)$ .

Interest payable per cycle =  $cI_pDT(T - M)$ .

Interest payable per year =  $cI_pD(T - M)$ .

(4) There are three cases to occur in interest earned per year.

Case I:  $0 < T < W/D$ , shown in Fig. 3.

According to assumption (6), the retailer can sell the items and earn interest with rate  $I_e$  throughout the inventory cycle.

Interest earned per cycle =  $sI_e \int_0^T Dt dt = \frac{DT^2sI_e}{2}$ .

Interest earned per year =  $\frac{DTsI_e}{2}$ .

Case II:  $W/D \leq T \leq M$ , shown in Fig. 4.

In this case, the retailer can sell the items and earn interest with rate  $I_e$  until the end of the trade credit period  $M$ .

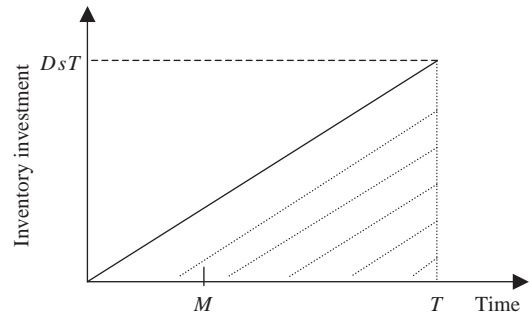


Fig. 3. The total accumulation of interest earned when  $0 < T < W/D$  or  $M \leq T$ .

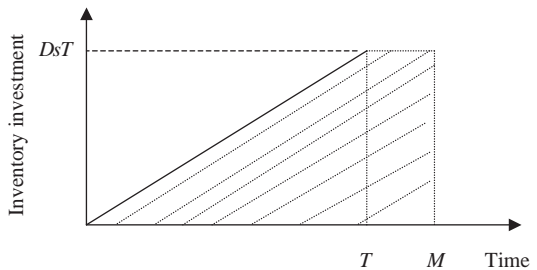


Fig. 4. The total accumulation of interest earned when  $T \leq M$ .

Interest earned per cycle =  $sI_e[\frac{DT^2}{2} + DT(M - T)] = DTsI_e(M - \frac{T}{2})$ .

Interest earned per year =  $DsI_e(M - \frac{T}{2})$ .

Case III:  $M \leq T$ , shown in Fig. 3.

In this case, the interest earned is similar to case I.

Interest earned per cycle =  $sI_e \int_0^T Dt dt = \frac{DT^2sI_e}{2}$ .

Interest earned per year =  $\frac{DTsI_e}{2}$ .

From the above, the total annual variable cost function for the retailer can be expressed as

$$\text{TVC}(T) = \text{ordering cost} + \text{stock-holding cost} + \text{interest payable} - \text{interest earned.}$$

We show that the annual total variable cost function,  $\text{TVC}(T)$ , is given by

$$\text{TVC}(T) = \text{TVC}_1(T) \quad \text{if } 0 < T < W/D, \tag{1a}$$

$$\text{TVC}(T) = \text{TVC}_2(T) \quad \text{if } W/D \leq T \leq M, \tag{1b}$$

$$\text{TVC}(T) = \text{TVC}_3(T) \quad \text{if } M \leq T, \tag{1c}$$

where

$$\text{TVC}_1(T) = \frac{A}{T} + \frac{DT h}{2} + cI_p D T - \frac{DT s I_e}{2}, \tag{2}$$

$$\text{TVC}_2(T) = \frac{A}{T} + \frac{DT h}{2} - D s I_e \left( M - \frac{T}{2} \right) \tag{3}$$

and

$$\text{TVC}_3(T) = \frac{A}{T} + \frac{DT h}{2} + cI_p D (T - M) - \frac{DT s I_e}{2}. \tag{4}$$

All  $\text{TVC}_1(T)$ ,  $\text{TVC}_2(T)$  and  $\text{TVC}_3(T)$  are defined on  $T > 0$ .  $\text{TVC}_1(W/D) > \text{TVC}_2(W/D)$  and  $\text{TVC}_2(M) = \text{TVC}_3(M)$ . Hence  $\text{TVC}(T)$  is well-defined and continuous except  $T = W/D$ . We also find  $\text{TVC}_1(T) > \text{TVC}_3(T)$  for all  $T > 0$ . Eqs. (2)–(4) yield

$$\text{TVC}'_1(T) = \frac{-A}{T^2} + \frac{D(h + 2cI_p - sI_e)}{2}, \tag{5}$$

$$\text{TVC}''_1(T) = \frac{2A}{T^3} > 0, \tag{6}$$

$$\text{TVC}'_2(T) = \frac{-A}{T^2} + \frac{D(h + sI_e)}{2}, \tag{7}$$

$$\text{TVC}''_2(T) = \frac{2A}{T^3} > 0, \tag{8}$$

$$\text{TVC}'_3(T) = \frac{-A}{T^2} + \frac{D(h + 2cI_p - sI_e)}{2} \tag{9}$$

and

$$\text{TVC}''_3(T) = \frac{2A}{T^3} > 0. \tag{10}$$

Eqs. (6), (8) and (10) imply that  $\text{TVC}_1(T)$ ,  $\text{TVC}_2(T)$  and  $\text{TVC}_3(T)$  are convex on  $T > 0$ .

(II) Suppose that  $W/D > M$ .

If  $W/D > M$ , Eqs. 1(a–c) will be modified as follows:

$$\text{TVC}(T) = \text{TVC}_1(T) \quad \text{if } 0 < T < W/D, \tag{11a}$$

$$\text{TVC}(T) = \text{TVC}_3(T) \quad \text{if } W/D \leq T. \tag{11b}$$

$\text{TVC}(T)$  is continuous except  $T = W/D$ .

### 3. Decision rule of the optimal cycle time when $M \geq W/D$

Recall

$$T_1^* = T_3^* = \sqrt{\frac{2A}{D(h + 2cI_p - sI_e)}} \tag{12}$$

if  $h + 2cI_p - sI_e > 0$

and

$$T_2^* = \sqrt{\frac{2A}{D(h + sI_e)}}. \tag{13}$$

Then  $\text{TVC}'_1(T_1^*) = \text{TVC}'_2(T_2^*) = \text{TVC}'_3(T_3^*) = 0$ . We also have the following result.

**Theorem 1.** (I) Suppose that  $h + 2cI_p < sI_e$ . Then  $T^* = \infty$ . (When  $T^* = \infty$ , it means that the retailer prefers to keep money and does not return money to the bank.)

(II) Suppose that  $h + 2cI_p = sI_e$ . Then

- (A) If  $T_2^* \geq M$ , then  $T^* = \infty$ .
- (B) If  $W/D \leq T_2^* < M$ , there are two cases to occur:
  - (a) If  $\text{TVC}_2(T_2^*) \leq -cI_p D M$ , then  $T^* = T_2^*$ .
  - (b) If  $\text{TVC}_2(T_2^*) > -cI_p D M$ , then  $T^* = \infty$ .
- (C) If  $T_2^* < W/D$ , there are two cases to occur:
  - (a) If  $\text{TVC}_2(W/D) \leq -cI_p D M$ , then  $T^* = W/D$ .
  - (b) If  $\text{TVC}_2(W/D) > -cI_p D M$ , then  $T^* = \infty$ .

**Proof.** (I) If  $h + 2cI_p < sI_e$ , Eq. (9) implies that  $\text{TVC}_3(T)$  is decreasing on  $T > 0$ . Hence Eqs. 1(a,

b, c) reveal that  $TVC(T)$  is decreasing on  $T \geq M$ . Since

$$\begin{aligned} \lim_{T \rightarrow \infty} TVC(T) &= \lim_{T \rightarrow \infty} TVC_3(T) \\ &= -cI_pDM + \lim_{T \rightarrow \infty} \frac{DT}{2}(h + 2cI_p - sI_e) \\ &= -\infty \end{aligned} \tag{14}$$

and

$$\begin{aligned} \lim_{T \rightarrow 0^+} TVC(T) &= \lim_{T \rightarrow 0^+} TVC_1(T) \\ &= \lim_{T \rightarrow 0^+} \left[ \frac{A}{T} + \frac{DT}{2}(h + 2cI_p - sI_e) \right] \\ &= \infty. \end{aligned} \tag{15}$$

Eqs. (14) and (15) imply  $T^* = \infty$ .

(II) (A) If  $h + 2cI_p = sI_e$  and  $T_2^* \geq M$ , then  $TVC_1(T)$ ,  $TVC_2(T)$  and  $TVC_3(T)$  are decreasing on  $(0, W/D)$ ,  $[W/D, M]$  and  $[M, \infty)$ , respectively. Hence,  $TVC(T)$  is decreasing on  $T > 0$ . Consequently  $T^* = \infty$ .

(B) If  $h + 2cI_p = sI_e$  and  $W/D \leq T_2^* < M$ , then Eqs. (5), (7) and (9) imply

- (i)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, T_2^*]$  and increasing on  $[T_2^*, M]$ .
- (iii)  $TVC_3(T)$  is decreasing on  $[M, \infty)$ .

Since  $\lim_{T \rightarrow \infty} TVC(T) = \lim_{T \rightarrow \infty} TVC_3(T) = -cI_pDM$  and  $TVC_2(T)$  has the minimum value at  $T = T_2^*$  on  $[W/D, M]$ , we have

- (a) If  $TVC_2(T_2^*) \leq -cI_pDM$ , then  $T^* = T_2^*$ .
- (b) If  $TVC_2(T_2^*) > -cI_pDM$ , then  $T^* = \infty$ .

(C) If  $h + 2cI_p = sI_e$  and  $T_2^* < W/D$ , then Eqs. (5), (7) and (9) imply

- (i)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .
- (ii)  $TVC_2(T)$  is increasing on  $[W/D, M]$ .
- (iii)  $TVC_3(T)$  is decreasing on  $[M, \infty)$ .

Since  $\lim_{T \rightarrow \infty} TVC(T) = \lim_{T \rightarrow \infty} TVC_3(T) = -cI_pDM$  and  $TVC_2(T)$  has the minimum value

at  $T = W/D$  on  $[W/D, M]$ , we have

- (a) If  $TVC_2(W/D) \leq -cI_pDM$ , then  $T^* = W/D$ .
- (b) If  $TVC_2(W/D) > -cI_pDM$ , then  $T^* = \infty$ .  $\square$

Based on Theorem 1, from now on, we assume  $h + 2cI_p > sI_e$ . Consequently,  $T_1^*$  and  $T_3^*$  are well-defined. By the convexity of  $TVC_i(T)$  ( $i = 1, 2, 3$ ), we see

$$TVC'_1(T) \begin{cases} < 0 & \text{if } T < T_1^*, \\ = 0 & \text{if } T = T_1^*, \\ > 0 & \text{if } T > T_1^*, \end{cases} \tag{16a}$$

$$= 0 \text{ if } T = T_1^*, \tag{16b}$$

$$> 0 \text{ if } T > T_1^*, \tag{16c}$$

$$TVC'_2(T) \begin{cases} < 0 & \text{if } T < T_2^*, \\ = 0 & \text{if } T = T_2^*, \\ > 0 & \text{if } T > T_2^* \end{cases} \tag{17a}$$

$$= 0 \text{ if } T = T_2^*, \tag{17b}$$

$$> 0 \text{ if } T > T_2^* \tag{17c}$$

and

$$TVC'_3(T) \begin{cases} < 0 & \text{if } T < T_3^*, \\ = 0 & \text{if } T = T_3^*, \\ > 0 & \text{if } T > T_3^*. \end{cases} \tag{18a}$$

$$= 0 \text{ if } T = T_3^*, \tag{18b}$$

$$> 0 \text{ if } T > T_3^*. \tag{18c}$$

Eqs. 16(a–c)–18(a–c) imply that  $TVC_i(T)$  is decreasing on  $(0, T_i^*)$  and increasing on  $[T_i^*, \infty)$  for all  $i = 1, 2, 3$ . Eqs. (5), (7) and (9) yield that

$$TVC'_1\left(\frac{W}{D}\right) = \frac{-2A + W^2/D(h + 2cI_p - sI_e)}{2(W/D)^2}, \tag{19}$$

$$TVC'_2\left(\frac{W}{D}\right) = \frac{-2A + W^2/D(h + sI_e)}{2(W/D)^2}, \tag{20}$$

$$TVC'_2(M) = \frac{-2A + DM^2(h + sI_e)}{2M^2} \tag{21}$$

and

$$TVC'_3(M) = \frac{-2A + DM^2(h + 2cI_p - sI_e)}{2M^2}. \tag{22}$$

Furthermore, we let

$$A_1 = -2A + \frac{W^2}{D}(h + 2cI_p - sI_e), \tag{23}$$

$$A_2 = -2A + \frac{W^2}{D}(h + sI_e), \tag{24}$$

$$A_3 = -2A + DM^2(h + sI_e) \tag{25}$$

and

$$\Delta_4 = -2A + DM^2(h + 2cI_p - sI_e). \tag{26}$$

Eqs. (23)–(26) yield that  $\Delta_4 \geq \Delta_1$  and  $\Delta_3 \geq \Delta_2$ . We also have

$$\Delta_1 > 0 \text{ if and only if } T_1^* < W/D, \tag{27}$$

$$\Delta_2 > 0 \text{ if and only if } T_2^* < W/D, \tag{28}$$

$$\Delta_3 > 0 \text{ if and only if } T_2^* < M, \tag{29}$$

$$\Delta_4 > 0 \text{ if and only if } T_3^* < M. \tag{30}$$

Therefore, we have the following results.

**Theorem 2.** *Suppose that  $h + 2cI_p > sI_e$ . Then*

- (A) *If  $\Delta_1 > 0, \Delta_2 \geq 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ , then  $TVC(T^*) = \min\{TVC(T_1^*), TVC(W/D)\}$ . Hence  $T^*$  is  $T_1^*$  or  $W/D$  associated with the least cost.*
- (B) *If  $\Delta_1 > 0, \Delta_2 < 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ , then  $TVC(T^*) = \min\{TVC(T_1^*), TVC(T_2^*)\}$ . Hence  $T^*$  is  $T_1^*$  or  $T_2^*$  associated with the least cost.*
- (C) *If  $\Delta_1 > 0, \Delta_2 < 0, \Delta_3 < 0$  and  $\Delta_4 \geq 0$ , then  $TVC(T^*) = \min\{TVC(T_1^*), TVC(M)\}$ . Hence  $T^*$  is  $T_1^*$  or  $M$  associated with the least cost.*
- (D) *If  $\Delta_1 \leq 0, \Delta_2 \geq 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ , then  $TVC(T^*) = TVC(W/D)$  and  $T^* = W/D$ .*
- (E) *If  $\Delta_1 \leq 0, \Delta_2 \geq 0, \Delta_3 \geq 0$  and  $\Delta_4 < 0$ , then  $TVC(T^*) = \min\{TVC(W/D), TVC(T_3^*)\}$ . Hence  $T^*$  is  $W/D$  or  $T_3^*$  associated with the least cost.*
- (F) *If  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ , then  $TVC(T^*) = TVC(T_2^*)$  and  $T^* = T_2^*$ .*
- (G) *If  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 \geq 0$  and  $\Delta_4 < 0$ , then  $TVC(T^*) = \min\{TVC(T_2^*), TVC(T_3^*)\}$ . Hence  $T^*$  is  $T_2^*$  or  $T_3^*$  associated with the least cost.*
- (H) *If  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 < 0$  and  $\Delta_4 \geq 0$ , then  $TVC(T^*) = TVC(M)$  and  $T^* = M$ .*
- (I) *If  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 < 0$  and  $\Delta_4 < 0$ , then  $TVC(T^*) = TVC(T_3^*)$  and  $T^* = T_3^*$ .*

**Proof.** (A) If  $\Delta_1 > 0, \Delta_2 \geq 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ . So  $TVC'_1(W/D) > 0, TVC'_2(W/D) \geq 0, TVC'_2(M) \geq 0$

and  $TVC'_3(M) \geq 0$ . Eqs. (27)–(30) imply that  $T_1^* < W/D, T_2^* \leq W/D, T_2^* \leq M$  and  $T_3^* \leq M$ , respectively. Furthermore, Eqs. 16(a–c)–18(a–c) imply that

- (i)  $TVC_3(T)$  is increasing on  $[M, \infty)$ .
- (ii)  $TVC_2(T)$  is increasing on  $[W/D, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, T_1^*]$  and increasing on  $[T_1^*, W/D)$ .

Combining (i)–(iii), we conclude that  $TVC(T)$  has the minimum value at  $T = T_1^*$  on  $(0, W/D)$  and  $TVC(T)$  has the minimum value at  $T = W/D$  on  $[W/D, \infty)$ . Hence

$$TVC(T^*) = \min\{TVC(T_1^*), TVC(W/D)\}.$$

Consequently,  $T^*$  is  $T_1^*$  or  $W/D$  associated with the least cost.

(B) If  $\Delta_1 > 0, \Delta_2 < 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ . So  $TVC'_1(W/D) > 0, TVC'_2(W/D) < 0, TVC'_2(M) \geq 0$  and  $TVC'_3(M) \geq 0$ . Eqs. (27)–(30) imply that  $T_1^* < W/D, T_2^* > W/D, T_2^* \leq M$  and  $T_3^* \leq M$ , respectively. Furthermore, Eqs. 16(a–c)–18(a–c) imply that

- (i)  $TVC_3(T)$  is increasing on  $[M, \infty)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, T_2^*]$  and increasing on  $[T_2^*, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, T_1^*]$  and increasing on  $[T_1^*, W/D)$ .

Combining (i)–(iii), we conclude that  $TVC(T)$  has the minimum value at  $T = T_1^*$  on  $(0, W/D)$  and  $TVC(T)$  has the minimum value at  $T = T_2^*$  on  $[W/D, \infty)$ . Hence

$$TVC(T^*) = \min\{TVC(T_1^*), TVC(T_2^*)\}.$$

Consequently,  $T^*$  is  $T_1^*$  or  $T_2^*$  associated with the least cost.

(C) If  $\Delta_1 > 0, \Delta_2 < 0, \Delta_3 < 0$  and  $\Delta_4 \geq 0$ . So  $TVC'_1(W/D) > 0, TVC'_2(W/D) < 0, TVC'_2(M) < 0$  and  $TVC'_3(M) \geq 0$ . Eqs. (27)–(30) imply that  $T_1^* < W/D, T_2^* > W/D, T_2^* > M$  and  $T_3^* \leq M$ , respectively. Furthermore, Eqs. 16(a–c)–18(a–c) imply that

- (i)  $TVC_3(T)$  is increasing on  $[M, \infty)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, T_1^*]$  and increasing on  $[T_1^*, W/D)$ .

Combining (i)–(iii), we conclude that  $TVC(T)$  has the minimum value at  $T = T_1^*$  on  $(0, W/D)$  and  $TVC(T)$  has the minimum value at  $T = M$  on  $[W/D, \infty)$ . Hence

$$TVC(T^*) = \min\{TVC(T_1^*), TVC(M)\}.$$

Consequently,  $T^*$  is  $T_1^*$  or  $M$  associated with the least cost.

(D) If  $\Delta_1 \leq 0, \Delta_2 \geq 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ . So  $TVC'_1(W/D) \leq 0, TVC'_2(W/D) \geq 0, TVC'_2(M) \geq 0$  and  $TVC'_3(M) \geq 0$ . Eqs. (27)–(30) imply that  $T_1^* \geq W/D, T_2^* \leq W/D, T_2^* \leq M$  and  $T_3^* \leq M$ , respectively. Furthermore, Eqs. 16(a–c)–18(a–c) imply that

- (i)  $TVC_3(T)$  is increasing on  $[M, \infty)$ .
- (ii)  $TVC_2(T)$  is increasing on  $[W/D, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Combining (i) and (ii) implies that  $TVC(T)$  has the minimum value at  $T = W/D$  on  $[W/D, \infty)$ . From (iii) and  $TVC_1(W/D) > TVC_2(W/D)$ , we conclude that  $TVC(T)$  has the minimum value at  $T = W/D$  on  $(0, \infty)$ . Hence  $T^* = W/D$ .

(E) If  $\Delta_1 \leq 0, \Delta_2 \geq 0, \Delta_3 \geq 0$  and  $\Delta_4 < 0$ . So  $TVC'_1(W/D) \leq 0, TVC'_2(W/D) \geq 0, TVC'_2(M) \geq 0$  and  $TVC'_3(M) < 0$ . Eqs. (27)–(30) imply that  $T_1^* \geq W/D, T_2^* \leq W/D, T_2^* \leq M$  and  $T_3^* > M$ , respectively. Furthermore, Eqs. 16(a–c)–18(a–c) imply that

- (i)  $TVC_3(T)$  is decreasing on  $[M, T_3^*]$  and increasing on  $[T_3^*, \infty)$ .
- (ii)  $TVC_2(T)$  is increasing on  $[W/D, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Combining (i) and (ii) yields that  $TVC(T)$  has the minimum value at  $T = W/D$  on  $[W/D, M]$  and  $TVC(T)$  has the minimum value at  $T = T_3^*$  on  $[M, \infty)$ . From (iii) and  $TVC_1(W/D) > TVC_2(W/D)$ , we conclude that

$$TVC(T^*) = \min\{TVC(W/D), TVC(T_3^*)\}.$$

Consequently,  $T^*$  is  $W/D$  or  $T_3^*$  associated with the least cost.

(F) If  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ . So  $TVC'_1(W/D) \leq 0, TVC'_2(W/D) < 0, TVC'_2(M) \geq 0$  and  $TVC'_3(M) \geq 0$ . Eqs. (27)–(30) imply that  $T_1^* \geq W/D, T_2^* > W/D, T_2^* \leq M$  and  $T_3^* \leq M$ ,

respectively. Furthermore, Eqs. 16(a–c)–18(a–c) imply that

- (i)  $TVC_3(T)$  is increasing on  $[M, \infty)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, T_2^*]$  and increasing on  $[T_2^*, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Since  $TVC_1(W/D) > TVC_2(W/D)$ , combining (i)–(iii) implies that  $TVC(T)$  has the minimum value at  $T = T_2^*$  on  $(0, \infty)$ . Consequently,  $T^* = T_2^*$ .

(G) If  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 \geq 0$  and  $\Delta_4 < 0$ . So  $TVC'_1(W/D) \leq 0, TVC'_2(W/D) < 0, TVC'_2(M) \geq 0$  and  $TVC'_3(M) < 0$ . Eqs. (27)–(30) imply that  $T_1^* \geq W/D, T_2^* > W/D, T_2^* \leq M$  and  $T_3^* > M$ , respectively. Furthermore, Eqs. 16(a–c)–18(a–c) imply that

- (i)  $TVC_3(T)$  is decreasing on  $[M, T_3^*]$  and increasing on  $[T_3^*, \infty)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, T_2^*]$  and increasing on  $[T_2^*, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Combining (i)–(iii), we conclude that  $TVC(T)$  has the minimum value at  $T = T_2^*$  on  $(0, M]$  and  $TVC(T)$  has the minimum value at  $T = T_3^*$  on  $[M, \infty)$ . Hence

$$TVC(T^*) = \min\{TVC(T_2^*), TVC(T_3^*)\}.$$

Consequently,  $T^*$  is  $T_2^*$  or  $T_3^*$  associated with the least cost.

(H) If  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 < 0$  and  $\Delta_4 \geq 0$ . So  $TVC'_1(W/D) \leq 0, TVC'_2(W/D) < 0, TVC'_2(M) < 0$  and  $TVC'_3(M) \geq 0$ . Eqs. (27)–(30) imply that  $T_1^* \geq W/D, T_2^* > W/D, T_2^* > M$  and  $T_3^* \leq M$ , respectively. Furthermore, Eqs. 16(a–c)–18(a–c) imply that

- (i)  $TVC_3(T)$  is increasing on  $[M, \infty)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Combining (i)–(iii), we conclude that  $TVC(T)$  has the minimum value at  $T = M$  on  $(0, \infty)$ . Consequently,  $T^* = M$ .

(I) If  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 < 0$  and  $\Delta_4 < 0$ . So  $TVC'_1(W/D) \leq 0, TVC'_2(W/D) < 0, TVC'_2(M) < 0$

and  $TVC'_3(M) < 0$ . Eqs. (27)–(30) imply that  $T_1^* \geq W/D$ ,  $T_2^* > W/D$ ,  $T_2^* > M$  and  $T_3^* > M$ , respectively. Furthermore, Eq. 16(a–c)–18(a–c) imply that

- (i)  $TVC_3(T)$  is decreasing on  $[M, T_3^*]$  and increasing on  $[T_3^*, \infty)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Combining (i)–(iii), we conclude that  $TVC(T)$  has the minimum value at  $T = T_3^*$  on  $(0, \infty)$ . Consequently,  $T^* = T_3^*$ .

Combining the above arguments, we have completed the proof of Theorem 2.  $\square$

**4. Decision rule of the optimal cycle time when  $M < W/D$**

When  $M < W/D$ ,  $TVC(T)$  can be expressed as follows:

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if } 0 < T < W/D, & (11a) \\ TVC_3(T) & \text{if } W/D \leq T. & (11b) \end{cases}$$

Then we have the following result.

**Theorem 3.** (I) Suppose that  $h + 2cI_p < sI_e$ . Then  $T^* = \infty$ .

(II) Suppose that  $h + 2cI_p = sI_e$ . Then  $T^* = \infty$ .

**Proof.** (I) See Theorem 1-(I).

(II) If  $h + 2cI_p = sI_e$ , Eqs. (5) and (9) imply that  $TVC_1(T)$  and  $TVC_3(T)$  is decreasing on  $(0, W/D)$  and  $[W/D, \infty)$ . Hence,  $TVC(T)$  is decreasing on  $(0, \infty)$ . Consequently,  $T^* = \infty$ .  $\square$

Eqs. (5) and (9) yield that

$$TVC'_1\left(\frac{W}{D}\right) = TVC'_3\left(\frac{W}{D}\right) = \frac{-2A + W^2/D(h + 2cI_p - sI_e)}{2(W/D)^2}$$

For convenience, we let  $\Delta = \Delta_1 = -2A + \frac{W^2}{D}(h + 2cI_p - sI_e)$ . Then we have the following result.

**Theorem 4.** Suppose that  $h + 2cI_p > sI_e$ . Then

(A) If  $\Delta > 0$ , then

$$TVC(T^*) = \min\{TVC(T_1^*), TVC(W/D)\}.$$

Hence  $T^*$  is  $T_1^*$  or  $W/D$  associated with the least cost.

- (B) If  $\Delta < 0$ , then  $TVC(T^*) = TVC(T_3^*)$  and  $T^* = T_3^*$ .
- (C) If  $\Delta = 0$ , then  $TVC(T^*) = TVC(W/D)$  and  $T^* = W/D$ .

**Proof.** (A) If  $\Delta > 0$ , then  $TVC'_1(W/D) = TVC'_3(W/D) > 0$ . Eqs. 16(a–c), 18(a–c) and (27) imply that

- (i)  $T_1^* = T_3^* < W/D$ .
- (ii)  $TVC_3(T)$  is increasing on  $[W/D, \infty)$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, T_1^*)$  and increasing on  $[T_1^*, W/D)$ .

Therefore,  $TVC(T)$  has the minimum value at  $T = T_1^*$  on  $(0, W/D)$  and  $TVC(T)$  has the minimum value at  $T = W/D$  on  $[W/D, \infty)$ . Hence

$$TVC(T^*) = \min\{TVC(T_1^*), TVC(W/D)\}.$$

Consequently,  $T^*$  is  $T_1^*$  or  $W/D$  associated with the least cost.

(B) If  $\Delta < 0$ , then

$$TVC'_1(W/D) = TVC'_3(W/D) < 0.$$

Eqs. 16(a–c), 18(a–c) and (27) imply that

- (i)  $T_1^* = T_3^* > W/D$ .
- (ii)  $TVC_3(T)$  is decreasing on  $[W/D, T_3^*]$  and increasing on  $[T_3^*, \infty)$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Combining (i)–(iii), we conclude that  $TVC(T)$  has the minimum value at  $T = T_3^*$  on  $(0, \infty)$ . Consequently,  $T^* = T_3^*$ .

(C) If  $\Delta = 0$ , then

$$TVC'_1(W/D) = TVC'_3(W/D) = 0.$$

Eqs. 16(a–c), 18(a–c) and (27) imply that

- (i)  $T_1^* = T_3^* = W/D$ .
- (ii)  $TVC_3(T)$  is increasing on  $[W/D, \infty)$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Since  $TVC_1(W/D) > TVC_3(W/D)$ , we conclude that  $TVC(T)$  has the minimum value at  $T = W/D$  on  $(0, \infty)$ . Consequently,  $T^* = W/D$ .



Combining the above arguments, we have completed the proof of Theorem 4. □

**5. The algorithm**

In this section, we shall combine Sections 3 and 4 to outline the algorithm to help us to decide the optimal cycle time and optimal order quantity.

**The algorithm**

Step 1: If  $M < W/D$ , then go to Step 5.

Step 2: If  $h + 2cI_p < sI_e$ , then  $T^* = \infty$ .

Step 3: If  $h + 2cI_p = sI_e$  and

- (i)  $T_2^* \geq M$ , then  $T^* = \infty$ .
- (ii)  $W/D \leq T_2^* < M$  if  $TVC_2(T_2^*) \leq -cI_pDM$ , then  $T^* = T_2^*$ . Otherwise,  $T^* = \infty$ .
- (iii)  $T_2^* < W/D$  if  $TVC_2(W/D) \leq -cI_pDM$ , then  $T^* = W/D$ . Otherwise,  $T^* = \infty$ .

Step 4: If  $h + 2cI_p > sI_e$  and

- (i)  $\Delta_1 > 0, \Delta_2 \geq 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ , then  $T^*$  is  $T_1^*$  or  $W/D$  associated with the least cost.
- (ii)  $\Delta_1 > 0, \Delta_2 < 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ , then  $T^*$  is  $T_1^*$  or  $T_2^*$  associated with the least cost.
- (iii)  $\Delta_1 > 0, \Delta_2 < 0, \Delta_3 < 0$  and  $\Delta_4 \geq 0$ , then  $T^*$  is  $T_1^*$  or  $M$  associated with the least cost.
- (iv)  $\Delta_1 \leq 0, \Delta_2 \geq 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ , then  $T^* = W/D$ .
- (v)  $\Delta_1 \leq 0, \Delta_2 \geq 0, \Delta_3 \geq 0$  and  $\Delta_4 < 0$ , then  $T^*$  is  $W/D$  or  $T_3^*$  associated with the least cost.
- (vi)  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 \geq 0$  and  $\Delta_4 \geq 0$ , then  $T^* = T_2^*$ .
- (vii)  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 \geq 0$  and  $\Delta_4 < 0$ , then  $T^*$  is  $T_2^*$  or  $T_3^*$  associated with the least cost.
- (viii)  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 < 0$  and  $\Delta_4 \geq 0$ , then  $T^* = M$ .
- (ix)  $\Delta_1 \leq 0, \Delta_2 < 0, \Delta_3 < 0$  and  $\Delta_4 < 0$ , then  $T^* = T_3^*$ .

Step 5: If  $h + 2cI_p \leq sI_e$ , then  $T^* = \infty$ .

Step 6: If  $h + 2cI_p > sI_e$  and

- (i)  $\Delta > 0$ , then  $T^*$  is  $T_1^*$  or  $W/D$  associated with the least cost.
- (ii)  $\Delta < 0$ , then  $T^* = T_3^*$ .
- (iii)  $\Delta = 0$ , then  $T^* = W/D$ .

**6. Special cases**

When  $W = 0$ , Eqs. 1(a–c) can be modified as follows:

$$TVC(T) = \begin{cases} TVC_2(T) & \text{if } 0 < T \leq M, \\ TVC_3(T) & \text{if } M \leq T. \end{cases} \quad (31a)$$

$$(31b)$$

Then Theorems 1 and 2 can be revised as Theorems 5 and 6, respectively.

**Theorem 5.** (I) Suppose that  $h + 2cI_p < sI_e$ . Then  $T^* = \infty$ .

(II) Suppose that  $h + 2cI_p = sI_e$ . Then

- (A) If  $T_2^* \geq M$ , then  $T^* = \infty$ .
- (B) If  $T_2^* < M$ , there are two cases to occur:
  - (a) If  $TVC_2(T_2^*) \leq -cI_pDM$ , then  $T^* = T_2^*$ .
  - (b) If  $TVC_2(T_2^*) > -cI_pDM$ , then  $T^* = \infty$ .

**Theorem 6.** Suppose that  $h + 2cI_p > sI_e$ . Then

- (A) If  $\Delta_4 \geq 0$  and  $\Delta_3 > 0$ , then  $T^* = T_2^*$ .
- (B) If  $\Delta_4 \geq 0$  and  $\Delta_3 \leq 0$ , then  $T^* = M$ .
- (C) If  $\Delta_4 < 0$  and  $\Delta_3 > 0$ , then  $T^*$  is  $T_2^*$  or  $T_3^*$  associated with the least cost.
- (D) If  $\Delta_4 < 0$  and  $\Delta_3 \leq 0$ , then  $T^* = T_3^*$ .

**7. Comparisons with Goyal’s model**

In this section, we assume that  $W = 0$  and  $s = c$ . Then  $h + 2cI_p > cI_e$ . Hence, Eqs. (12) and (13) can be rewritten as

$$T_2^* = \sqrt{\frac{2A}{D(h + cI_e)}} \quad (32)$$

and

$$T_3^* = \sqrt{\frac{2A}{D(h + 2cI_p - cI_e)}}. \quad (33)$$

Furthermore, Eqs. (25) and (26) can be reduced to

$$\Delta_3 = -2A + DM^2(h + cI_e) \quad (34)$$

and

$$\Delta_4 = -2A + DM^2(h + 2cI_p - cI_e). \quad (35)$$

Eqs. (32)–(35) yield that  $\Delta_4 \geq \Delta_3$  and  $T_3^* \leq T_2^*$ . Then Theorem 2 yields the following result.

**Theorem 7.** Suppose that  $W = 0$  and  $s = c$ . Then

- (A) If  $\Delta_4 \geq \Delta_3 > 0$ , then  $T^* = T_2^*$ .
- (B) If  $\Delta_4 \geq \Delta_3 = 0$ , then  $T^* = M$ .
- (C) If  $\Delta_4 \geq 0$  and  $\Delta_3 < 0$ , then  $T^* = M$ .
- (D) If  $\Delta_4 < 0$  and  $\Delta_3 < 0$ , then  $T^* = T_3^*$ .

Let  $\bar{T}_3^* = \sqrt{\frac{2A + DcM^2(I_p - I_e)}{D(h + cI_p)}}$  and  $\bar{T}_2^* = \sqrt{\frac{2A}{D(h + cI_e)}}$ . Moreover, we let  $\bar{T}^*$  denote the optimal cycle time of Goyal’s model (1985).

Theorem 1 in Chung (1998a) determines the optimal cycle time of Goyal’s model (1985) can be described as follows:

- Theorem 8.** (A) If  $\Delta_3 > 0$ , then  $\bar{T}^* = \bar{T}_2^*$ .
- (B) If  $\Delta_3 < 0$ , then  $\bar{T}^* = \bar{T}_3^*$ .
- (C) If  $\Delta_3 = 0$ , then  $\bar{T}^* = \bar{T}_2^* = \bar{T}_3^* = M$ .

Then we have the following result.

**Theorem 9.**  $T^* \leq \bar{T}^*$ . In fact, we have

- (A) If  $\Delta_4 \geq \Delta_3 > 0$ , then  $T^* = \bar{T}^* = T_2^* = \bar{T}_2^*$ .
- (B) If  $\Delta_4 \geq \Delta_3 = 0$ , then  $T^* = \bar{T}^* = M$ .
- (C) If  $\Delta_4 \geq 0$  and  $\Delta_3 < 0$ , then  $T^* < \bar{T}^*$ .
- (D) If  $\Delta_4 < 0$  and  $\Delta_3 < 0$ , then  $T^* \leq \bar{T}^*$ .

**Proof.** (A) If  $\Delta_4 \geq \Delta_3 > 0$ , then both Theorems 7 and 8 imply  $T^* = T_2^*$  and  $\bar{T}^* = \bar{T}_2^*$ . However  $T_2^* = \bar{T}_2^*$ . Hence  $T^* = \bar{T}^* = T_2^* = \bar{T}_2^*$ .

(B) If  $\Delta_4 \geq \Delta_3 = 0$ , then both Theorems 7 and 8 imply  $T^* = M$  and  $\bar{T}^* = M$ . Hence  $T^* = \bar{T}^* = M$ .

(C) If  $\Delta_4 \geq 0$  and  $\Delta_3 < 0$ , then both Theorems 7 and 8 imply  $T^* = M$  and  $\bar{T}^* = \bar{T}_3^*$ . Since

$$\begin{aligned} (\bar{T}^*)^2 - (T^*)^2 &= \frac{2A + DcM^2(I_p - I_e)}{D(h + cI_p)} - M^2 \\ &= \frac{2A - DM^2(h + cI_e)}{D(h + cI_p)} \\ &= \frac{-\Delta_3}{D(h + cI_p)} > 0, \end{aligned}$$

we have  $T^* < \bar{T}^*$ .

(D) If  $\Delta_4 < 0$  and  $\Delta_3 < 0$ , then both Theorems 7 and 8 imply  $T^* = T_3^*$  and  $\bar{T}^* = \bar{T}_3^*$ . Since

$$\begin{aligned} (\bar{T}^*)^2 - (T^*)^2 &= \frac{2A + DcM^2(I_p - I_e)}{D(h + cI_p)} - \frac{2A}{D(h + 2cI_p - cI_e)} \\ &= \frac{[2A + DcM^2(I_p - I_e)](h + 2cI_p - cI_e) - 2A(h + cI_p)}{D(h + cI_p)(h + 2cI_p - cI_e)} \\ &= \frac{DcM^2(I_p - I_e)(h + 2cI_p - cI_e) + 2Ac(I_p - I_e)}{D(h + cI_p)(h + 2cI_p - cI_e)} \geq 0, \end{aligned}$$

we have  $T^* \leq \bar{T}^*$ .

Combining the above arguments, we have completed the proof of Theorem 9.  $\square$

Theorem 9 explains that the optimal cycle time when  $W = 0$  and  $s = c$  is not longer than that of Goyal’s model (1985).

### 8. Summary

This article discusses the economic order quantity under conditions of permissible delay in payments to take the order quantity into account. If  $Q < W$ , the delay in payments is not permitted. Otherwise, the fixed trade credit period  $M$  is permitted. There are two cases (i)  $M \geq W/D$  and (ii)  $M < W/D$  to be explored. Theorems 1 and 2 give the solution procedure to find  $T^*$  when  $M \geq W/D$ . Theorems 3 and 4 give the solution procedure to find  $T^*$  when  $M < W/D$ . Then, we develop an algorithm to help us to decide  $T^*$ . Furthermore, Theorems 5 and 6 reveal the solution procedure to find  $T^*$  when  $W = 0$ . Finally, when  $W = 0$  and  $s = c$ , this article develops some comparisons with Goyal’s model (1985) and demonstrate that the optimal cycle time is not longer than that of Goyal’s model (1985).

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